

Consistent Probabilistic Description of the Neutral Kaon System: Novel Observable Effects

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Abstract

We describe the neutral Kaon system as an open Lindblad-type quantum mechanical system due to Kaon decays. Including CP violation in the mixing, the formalism allows for total probability conservation, in contrast to the standard Weisskopf-Wigner Approach (WWA) employing a non-hermitian Hamiltonian to integrate out the decay products. This approach, in terms of density matrices for initial and final states, provides a consistent probabilistic treatment. Even for an initial pure state, the time evolution generates a mixed state, not described by a projector. To restore Unitarity, we include the dominant decay channel to two pions, so that one of the Kaon states with definite lifetime becomes stable. The new dynamics for initial and final states modifies the observables to second order in the CP-violating parameter ϵ , as compared to WWA.

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Neutral Kaons is a fascinating physical system that, due to its peculiar and at the time paradoxical behaviour in many respects, has lead to important discoveries, thereby triggering an enormous interest for its study. It is the first physical system where CP violation has been observed in the two-pion $K^0 \rightarrow 2\pi$ decay channel [1], with the relevant experimental studies continuing up to date [2–4] and extended to entangled neutral Kaon states in meson ϕ factories [5]. Moreover, neutral Kaons have also been used as a probe of fundamental symmetries, such as CPT invariance [6], and deviations from the standard quantum mechanical behaviour. The latter may be induced by quantum gravity fluctuations appearing as a “decoherening” environment, leading to an open system (Lindblad-type [7]) formulation [8–12].

The standard description of the neutral-Kaon system follows the Weisskopf-Wigner Approach (WWA) [13] for unstable particles using the non-Hermitian Hamiltonian

$$\hat{\mathcal{H}} = \hat{\mathcal{M}} - i\frac{\hat{\Gamma}}{2}. \quad (1)$$

However, the simultaneous presence of CP violation in the mass matrix $\hat{\mathcal{M}}$ and a difference of lifetimes in the antihermitian matrix $i\hat{\Gamma}/2$ leads to a quantum incompatibility between $\hat{\mathcal{M}}$ and $\hat{\Gamma}$, $[\hat{\mathcal{M}}, \hat{\Gamma}] \neq 0$, *i.e.*, one cannot define states of definite mass and lifetime simultaneously, because $\hat{\mathcal{H}}$ is not a normal operator. The eigenstates K_L and K_S , obtained by a non-unitary diagonalization of $\hat{\mathcal{H}}$, lack physical meaning and their non-orthogonality prevents a consistent probabilistic treatment of this and any

other system with a Hamiltonian which is an abnormal operator.

The WWA, restricted to the description of the dynamics of the initial two-state system, is furthermore unsatisfactory from the point of view of securing Unitarity, *i.e.*, conservation of probability. The latter property has to be imposed externally to the formalism, via the Bell-Steinberger Unitarity relation [14]. For this purpose, the inclusion of the $\pi\pi$ final state in the neutral-Kaon decays constitutes an excellent approximation towards restoration of Unitarity.

The lack of a proper probabilistic interpretation of the neutral Kaon system has been addressed previously [15, 16] by distinguishing the ket and bra Hilbert spaces, so that the lack of orthogonality of the K_L , K_S states is bypassed. The treatment is, however, limited to considering the dynamics of pure initial states, without the inclusion of the final ones nor the evolution to mixed states.

In [17, 18] a suggestion has been made to view a decaying quantum system as an open system interacting with an appropriate “environment” obtained by enlarging the original Hilbert space by states representing the decay products. The time evolution of such a system can be described by an effective hermitian Hamiltonian, essentially $\hat{\mathcal{M}}$ above, and an additional dissipative term of Lindblad form (*dissipator*) [7]. As shown in [18], the non-hermitian part of the Hamiltonian in the WWA, associated with the particle decay width operator $\hat{\Gamma}$, can be incorporated into the dissipator of the enlarged space

via a specific Lindblad operator \mathcal{B} .

To understand in simple terms the logic behind this open-quantum-system formalism for decaying systems, we first concentrate our attention on the evolution equation for the initial density matrix, $\rho = |\Psi\rangle\langle\Psi|$:

$$\dot{\rho} = -i\hat{\mathcal{H}}\rho + i\rho\hat{\mathcal{H}}^\dagger = -i\left[\hat{\mathcal{M}}, \rho\right] - \frac{1}{2}\left\{\rho, \hat{\Gamma}\right\}, \quad (2)$$

where $\dot{\rho}$ denotes time derivative. This equation can be *formally* obtained from the Schrödinger equation for the state vector $|\Psi\rangle$ with the *non-Hermitian* Hamiltonian \mathcal{H} . In eq. (2), $\hat{\Gamma}$ is viewed as a single quantum mechanical operator. The non-Hermiticity of \mathcal{H} leads to the anti-commutator term in the right-hand-side of this evolution equation. As a consequence, the description of the system in terms of pure states, for which $\text{Tr}\rho^2 = \text{Tr}\rho$, breaks down at time $t > 0$. This can be readily shown by calculating the rate of the Von-Neumann entropy $S = -\text{Tr}(\rho \ln \rho)$ using the evolution (2), where only the anticommutator part contributes. Not only the system is evolving with time to mixed states, but $\text{Tr}\dot{\rho}(t) \neq 0$, so that the restriction to the initial Hilbert space, ignoring the decay products, leads to violation of Unitarity with non-conservation of Probability.

To restore Unitarity we must include the final states when taking the trace $\text{Tr}\rho$ through a mapping from the initial Hilbert space to the final one (decay products): $H_i \rightarrow H_f$. This mapping is implemented [18] by the transition operator \mathcal{B} , which is related to $\hat{\Gamma}$ via:

$$\mathcal{B}^\dagger \mathcal{B} = \hat{\Gamma}. \quad (3)$$

If $\{f_k\}$ denotes an orthonormal basis in H_f , and $\{\varphi_j\}$ denotes the corresponding orthonormal basis in H_i (orthogonal to $\{f_k\}$), then one may write:

$$\mathcal{B} = \sum_{k=1}^{d_f} \sum_{j=1}^{d_i} b_{kj} |f_k\rangle\langle\varphi_j|, \quad (4)$$

where $d_f = \dim H_f$, and $d_i = \dim H_i$. The width operator $\hat{\Gamma}$ is thus a positive definite self-adjoint operator with non-negative eigenvalues. The latter can include possible zero eigenvalues, corresponding to stable states. This is to be contrasted to the corresponding expression given in [18] and will have important consequences for the neutral-Kaon system.

The operator \mathcal{B} can be considered as a sort of “*environment*” operator from the point of view of the initial state Hilbert space, and the evolution (2) can be replaced now by an appropriate Lindblad evolution [7], with ρ spanning the combined initial (H_i) and final (H_f) Hilbert spaces, $H_{tot} \equiv H_i \oplus H_f$. The Lindblad evolution can be understood as follows: in view of (3), the simple commutator structure of (2) in the conventional WWA [10] will now be replaced by an appropriate *quantum* ordering of the constituent operators $\mathcal{B}, \mathcal{B}^\dagger$ and ρ

in such a way that the time evolution has the following properties [7]: (i) preserves the complete positivity of the density matrix operators at any time, *i.e.*, the fact that their eigenvalues are positive or zero, so that the concept of probabilities associated with the eigenvalues of these operators makes sense, (ii) ensures the conservation of the total probability through $\text{Tr}\rho = 1$, including the *final* states (decay products) and (iii) implies increase of the entropy (of quantum states).

Our density matrix ρ in the total Hilbert space $H_{tot} \equiv H_i \oplus H_f$ is: $\rho = \begin{pmatrix} \rho_{ii'} & \rho_{if} \\ \rho_{fi} & \rho_{ff'} \end{pmatrix}$, where Hermiticity of ρ is fulfilled in blocks and the subindices ii' (ff') run over the initial (final) states. We have for the dimension of the relevant Hilbert spaces:

$$\dim H_i < \infty, \quad \text{and} \quad \dim H_f \geq r = \dim H_i - n_0, \quad (5)$$

with n_0 the degeneracy of the eigenvalue zero of the width operator. The evolution equations for the density matrix ρ in the H_{tot} Hilbert space are then described by the Lindblad form [7]:

$$\dot{\rho} = -i[\mathcal{H}, \rho] - \frac{1}{2}(\mathcal{B}^\dagger \mathcal{B} \rho + \rho \mathcal{B}^\dagger \mathcal{B} - 2\mathcal{B} \rho \mathcal{B}^\dagger), \quad (6)$$

with

$$\mathcal{H} = \mathcal{H}^\dagger = \begin{pmatrix} \hat{\mathcal{M}} & 0 \\ 0 & 0 \end{pmatrix}, \quad \mathcal{B} = \begin{pmatrix} 0 & 0 \\ \mathcal{B} & 0 \end{pmatrix}, \quad (7)$$

in total Hilbert space. The new formulation of the time evolution on the enlarged space has a hermitian Hamiltonian and is probability conserving. The complete positivity, that is guaranteed by construction in the Lindblad formalism [7], ensures that this feature characterises the decaying quantum system, exactly as it happens in systems with Hermitian Hamiltonians.

We would like to discuss here the application of this Lindblad open-system formulation of particle decay to physically realistic systems, such as neutral Kaons, which are known to exhibit CP violation and non-zero width difference $\Delta\Gamma \neq 0$. Contrary to WWA and the dynamics given by eq. (2), the open-system formalism is applicable in terms of the transition operator \mathcal{B} (7), irrespective of the commutativity of the composite $\hat{\Gamma}$ operator with $\hat{\mathcal{M}}$. In this respect, the Lindblad dynamics (6) for the decay is appropriate mainly for neutral Kaons. Other neutral mesons, such as $B-\bar{B}$ systems, are characterised by very small width differences between the physical eigenstates, practically $\Delta\Gamma \simeq 0$, for which the non-Hermitian Hamiltonian is a normal operator and the WWA is satisfactory.

For this discussion, we focus our attention from now

on two-state unstable systems. We can write eq. (6) as :

$$\begin{pmatrix} \dot{\rho}_{ii'} & \dot{\rho}_{if} \\ \dot{\rho}_{fi} & \dot{\rho}_{ff'} \end{pmatrix} = \begin{pmatrix} -i[\widehat{\mathcal{M}}, \rho_{ii'}] - \frac{1}{2}\{\widehat{\Gamma}, \rho_{ii'}\} & -i\widehat{\mathcal{M}}\rho_{if} - \frac{1}{2}\widehat{\Gamma}\rho_{if} \\ i\rho_{fi}\widehat{\mathcal{M}} - \frac{1}{2}\rho_{fi}\widehat{\Gamma} & \mathcal{B}\rho_{ii'}\mathcal{B}^\dagger \end{pmatrix}. \quad (8)$$

One notes the following: (i) the new dynamical behaviour of the final state coupled to the initial one, with effects which cannot be described by $\widehat{\Gamma}$ only; (ii) the formally identical structure of the equation for the time evolution of the initial-state density matrix, which is uncoupled to the final states, to that of eq. (2), as a result of the anticommutator $\{\widehat{\Gamma}, \rho_{ii'}\}$; however, here the evolution of pure to mixed states, in the sense of $\text{Tr}\rho(t)^2 \neq \text{Tr}\rho(t)$ at a time $t > 0$, is evident due to the effects of the Lindblad operator \mathcal{B} ; (iii) the uncoupled dynamical behaviour of $\rho_{if}(t)$, so that it is consistent to take $\rho_{if}(t) = 0$, if there is no initial ($t = 0$) mixed component between initial and final states; this implies that the description of the time evolution and decay is expressed in terms of initial and final density matrices only.

The reader should recall once more that in the Lindblad approach, total probability conservation for the density matrix, including the decay products, is guaranteed by construction, *i.e.*, $\text{Tr}\rho_{ii} + \text{Tr}\rho_{ff} = 1$ for any t , so that Unitarity is implied by the simple relation: $\frac{d\text{Tr}(\rho_{ii}(t))}{dt} = -\frac{d\text{Tr}(\rho_{ff}(t))}{dt}$. This relation can be verified explicitly for the solutions we obtain here for the case of the neutral Kaon system.

For the neutral Kaon $K^0 - \bar{K}^0$ system we incorporate properly CP violation and the dynamics of its dominant decay to two pion final states. We use the $|K_{1,2}\rangle$ basis for the initial Kaon states defined as:

$$|K_1\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle - |\bar{K}^0\rangle), \quad |K_2\rangle = \frac{1}{\sqrt{2}}(|K^0\rangle + |\bar{K}^0\rangle), \quad (9)$$

which, as we show below, is a convenient choice in which the width operator is diagonal.

The existence of a *dominant* decay channel in the neutral Kaon system, implies, via eq. (5), that $n_0 = 1$, which is correct, given that there is only one zero eigenvalue in the spectrum of $\widehat{\Gamma}$. It would correspond to the K_L state in the WWA within time scales of order of the K_S lifetime. Ignoring CPT Violation and CP violation in the decay, the choice of a real \mathcal{B} leads to the result that the $K_{1,2}$ states are the ones with definite lifetimes, so that the width operator in the $|K_{1,2}\rangle$ basis is given by the following 2×2 diagonal matrix with eigenvalues 0 and Γ_S :

$$\widehat{\Gamma}_{\text{WWA}} = \begin{pmatrix} \Gamma - \text{Re}(\Gamma_{12}) & 0 \\ 0 & \Gamma + \text{Re}(\Gamma_{12}) \end{pmatrix} = \Gamma_S \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}, \quad (10)$$

In this case the Lindblad operator (6), related to $\widehat{\Gamma}$ via (3), is given by the following row matrix:

$\mathcal{B} = \sqrt{\Gamma_S}(0, 1)$. In the $|K_{1,2}\rangle$ basis (9), the mass $\widehat{\mathcal{M}}$ matrix, which will play the rôle of the Hermitian Hamiltonian, is written as [10]:

$$\widehat{\mathcal{M}} = \begin{pmatrix} M - \text{Re}(M_{12}) & -i\text{Im}(M_{12}) \\ i\text{Im}(M_{12}) & M + \text{Re}(M_{12}) \end{pmatrix}, \quad (11)$$

ignoring again possible CPT-Violating effects. The CP violation parameter ϵ is given by:

$$\epsilon = |\epsilon| e^{-i\phi} = \frac{\text{Im}(M_{12})}{\frac{\Gamma_S}{2} + i\Delta m}, \quad \tan \phi = \frac{2\Delta m}{\Gamma_S}, \quad (12)$$

where $\Delta m = 2|M_{12}|$ is the difference between the mass eigenvalues ($\lambda_{1,2} = M \mp |M_{12}|$) of the Kaon mass eigenstates. The latter are found to differ from the $K_{1,2}$ states by terms of order of the CP violation parameter ϵ : $|v_1\rangle = |K_1\rangle + \mathcal{O}(\epsilon)|K_2\rangle$, $|v_2\rangle = |K_2\rangle + \mathcal{O}(\epsilon)|K_1\rangle$. As this transformation between life-time and mass eigenstates is unitary [19], the *orthogonality* of the latter is guaranteed. This is in contrast to the WWA K_L and K_S states.

To solve the evolution equations (8), we shall follow the analysis in [10], and use a perturbation method, by which we expand the density matrix elements at any time t in powers of the amplitude of the small CP-violation parameter $|\epsilon|$ (12): $\rho_{IJ}(t) = \rho_{IJ}^{(0)}(t) + \rho_{IJ}^{(1)}(t) + \rho_{IJ}^{(2)}(t) + \dots, \rho_{IJ}^{(n)}(t) \propto |\epsilon|^n$, $n = 0, 1, 2, \dots$, where the indices I, J span the full Hilbert space of states $\{i, f\}$, including the decay products (final) states.

In our analysis we shall restrict ourselves to second order in $|\epsilon|$, which matches the currently expected experimental sensitivity. From equations (8), first one solves the evolution equation for the initial states $\rho_{ii'}(t)$, $i, i' = \{1, 2\}$, to order $|\epsilon|^2$ and then obtain $\dot{\rho}_{ff}(t)$, associated with the $f = (\pi, \pi)$ decay channel, through $\dot{\rho}_{ff}(t) = \Gamma_S \rho_{22}(t)$. The result, expressed in terms of the initial conditions for $\rho_{ii'}(0)$, reads:

$$\begin{aligned} \rho_{22}(t) &= \rho_{22}(0)e^{-\Gamma_S t} \\ &- 2|\epsilon||\rho_{12}(0)| \left[e^{-\Gamma_S t} \cos(\phi + \phi_{12}) - e^{-\frac{\Gamma_S}{2}t} \cos(\Delta m t - \phi - \phi_{12}) \right] \\ &+ |\epsilon|^2 \left[\rho_{11}(0) + e^{-\Gamma_S t} \left(\rho_{11}(0) + \rho_{22}(0)(2\cos(2\phi) + \Gamma_S t) \right) \right] \\ &- 2e^{-\frac{\Gamma_S}{2}t} \left(\rho_{11}(0) \cos(\Delta m t) + \rho_{22}(0) \cos(\Delta m t - 2\phi) \right) \end{aligned} \quad (13)$$

with $\phi_{12} = \text{Arg}\rho_{12}(0)$.

We can use this result in order to calculate various observables of the Kaon system, in the above approximation of non-decaying K_1 state. We can build useful observables associated to the decay to $\pi\pi$ or semileptonic decays $\pi l \nu$. For a complete set of observables we refer the reader to a forthcoming publication [19]. For our purposes here we shall concentrate on two specific observables, namely the decay rates $R(K^0 \rightarrow \pi\pi)$ and $R(\bar{K}^0 \rightarrow \pi\pi)$, which will be compared to the corresponding observables in the WWA formalism, under the approximation $\Gamma_L = 0$, which is a good one for the range of times we consider. To be more specific, we will construct separately the sum of

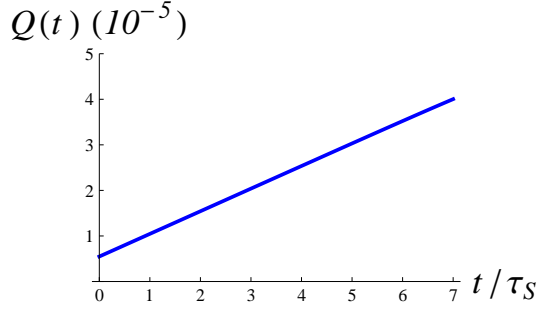


FIG. 1: The quantity $Q(t) \equiv \frac{\Sigma R}{\Sigma R|_{\text{WWA}}} - 1$ versus time (in units of the life-time τ_S), where ΣR is given in eq. (15). The suffix WWA indicates the corresponding quantities in the WWA.

rates, sensitive to even powers of $|\epsilon|$, and their difference (or CP-violating asymmetry), sensitive to odd powers of $|\epsilon|$.

To this end we need the initially pure K^0 and \bar{K}^0 states, prepared experimentally, in the $|K_{1,2}\rangle$ basis. These are described at $t = 0$ in the total Hilbert space H_{tot} by the density matrices:

$$\rho_{K^0} = \frac{1}{2} \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \rho_{\bar{K}^0} = \frac{1}{2} \begin{pmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}. \quad (14)$$

To order $|\epsilon|^2$ we obtain:

$$\begin{aligned} \Delta R &\equiv R(K^0 \rightarrow \pi\pi) - R(\bar{K}^0 \rightarrow \pi\pi) = \\ &-2|\epsilon|\Gamma_S e^{-t\Gamma_S} (\cos\phi - e^{\frac{\Gamma_S}{2}t} \cos(\Delta mt - \phi)) \\ \Sigma R &\equiv R(\bar{K}^0 \rightarrow \pi\pi) + R(K^0 \rightarrow \pi\pi) = \\ &\Gamma_S e^{-t\Gamma_S} \left[1 + |\epsilon|^2 (1 + e^{t\Gamma_S} + t\Gamma_S + \right. \\ &\left. 2\cos(2\phi) - 4e^{\frac{\Gamma_S}{2}t} \cos(\Delta mt - \phi) \cos\phi) \right]. \end{aligned} \quad (15)$$

Comparing with the WWA formalism we observe that the quantities (15) differ from the corresponding expressions in WWA [2] by terms of order $|\epsilon|^2$. The CP-violating decay-rate difference, ΔR , on the other hand, is of order $|\epsilon|$, agrees with the WWA result. In FIG. 1 we plot the difference from unity of the ratio of the sum of the rates for our result (15) over that of the WWA as a function of time. We observe that differences of order 10^{-5} occur already at times of order $5\tau_S$, which makes the experimental detection of the corrections challenging.

For those readers concerned by the possible competition of these novel effects of order $|\epsilon|^2$ with those coming from direct CP violation or order ϵ' we point out the following: (i) The difference $Q(t)$ is time dependent, (ii) what we call “ $(\pi\pi)$ ” in this work denotes the combination of rates $\frac{1}{3}[2(\pi^+\pi^-) + (\pi^0\pi^0)]$, in which the contributions linear in ϵ' cancel out.

To conclude, we have presented a description of the decaying neutral Kaon system as an open Lindblad system

involving evolution of pure to mixed states. It satisfies all the physical requirements of a probabilistic quantum mechanical interpretation and guarantees Unitarity, provided that the width operator in the dynamics of the initial states is a composite operator expressed in terms of the transition operator \mathcal{B} between initial and final Hilbert spaces. As a consequence, the time dependence of the observable decay rates is modified, with respect to Weisskopf-Wigner Approach, to quadratic order in the magnitude of the CP violating parameter $|\epsilon|^2$. In our view, the measurement of these novel observable effects constitute a new experimental challenge in the field.

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